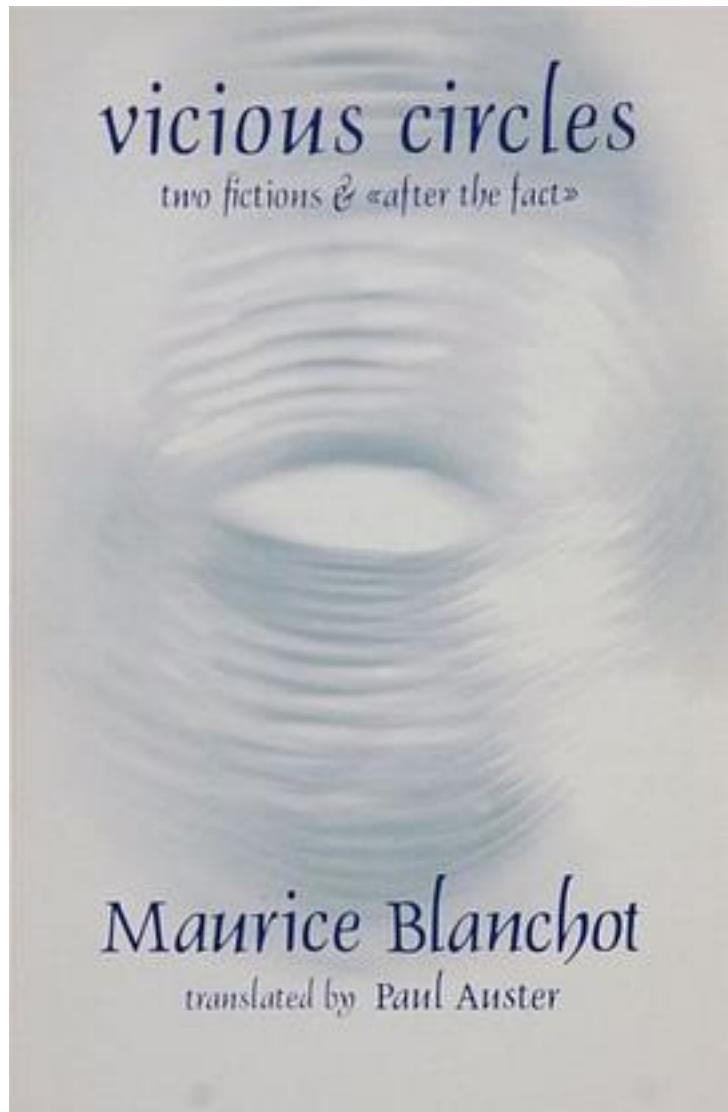


# Vicious Circles



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Many assume that circular phenomena and mathematical rigour are irreconcilable. Barwise and Moss have undertaken to prove this assumption false. *Vicious Circles* is intended for use by researchers who use hypersets, although the book is accessible to people with widely differing backgrounds and interests.

The following is a comment from amazon.com

This book discusses recent advances in the general field of set theory. The authors study a variant of ZF in which the axiom of foundation is replaced by a new axiom allowing non-well-founded sets. Just as the naturals can be extended to the integers, and the integers to the rationals, and the reals to the complex numbers, in each case by positing new numbers that are the solutions to a class of equations, so this book posits an extension to any model of set theory consisting of the solutions to a class of (systems of) equations having no solutions in ZF. The simplest example is the equation

$$x = \{x\},$$

whose solution,

$$x = \{\{\{\dots\}\}\} \text{ (infinitely deep)}$$

is not permitted in ZF, but exists and is unique in the authors' theory.

The purpose of this extension to ZF is to create a set theory in which certain circular or infinite phenomena from computer science and other fields, e.g. cyclic data streams, can be much more directly modeled than is now possible in ZF. Currently in ZF in order to represent a cyclic data stream one has to develop the apparatus for natural numbers, and then represent the stream to be a function from the natural numbers into some suitable set representing the type of data. But in the author's set theory the stream could be represented as an unfounded set that is the solution to a simple equation, and many of its properties could then be more easily deduced without resort to arithmetic.

I found this book absolutely fascinating, and I highly recommend it to anyone who has had a course in set theory. The theory in the book is quite elegant and satisfying.

I was delighted to learn that there is still room for new variations of the axioms of set theory, a subject I thought (probably naively) had been fairly static for 60 years.

作者介绍:

Jon Barwise (1942–2000) was professor of philosophy, mathematics, and computer science at Indiana University and one of the founding members of the Center for the Study of Language and Information (CSLI).

Lawrence S. Moss is professor of mathematics; director of the Program in Pure and Applied Logic; an adjunct professor of computer science, informatics, linguistics, and philosophy; and a member of the Programs in Cognitive Science and Computational Linguistics, all at Indiana University, Bloomington.

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