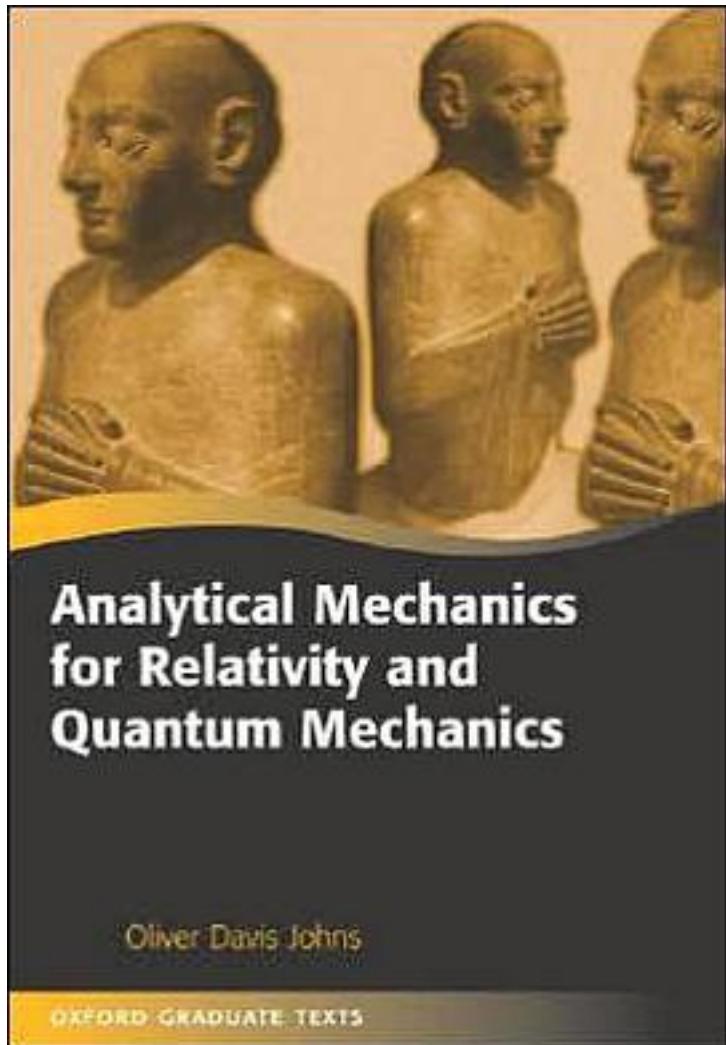


# Analytical Mechanics for Relativity and Quantum Mechanics



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This book provides an innovative and mathematically sound treatment of the foundations of analytical mechanics and the relation of classical mechanics to relativity and quantum theory. It is intended for use at the introductory graduate level. A distinguishing feature of the book is its integration of special relativity into teaching of classical mechanics. After a thorough review of the traditional theory, Part II of the book introduces extended Lagrangian and Hamiltonian methods that treat time as a transformable coordinate rather than the fixed parameter of Newtonian physics. Advanced topics such as covariant Langrangians and Hamiltonians, canonical transformations, and Hamilton-Jacobi methods are simplified by the use of this extended theory. And the definition of canonical transformation no longer excludes the Lorenz transformation of special relativity. This is also a book for those who study analytical mechanics to prepare for a critical exploration of quantum mechanics. Comparisons to quantum mechanics appear throughout the text. The extended Hamiltonian theory with time as a coordinate is compared to Dirac's formalism of primary phase space constraints. The chapter on relativistic mechanics shows how to use covariant Hamiltonian theory to write the Klein-Gordon and Dirac equations. The chapter on Hamilton-Jacobi theory includes a discussion of the closely related Bohm hidden variable model of quantum mechanics. Classical mechanics itself is presented with an emphasis on methods, such as linear vector operators and dyadics, that will familiarize the student with similar techniques in quantum theory. Several of the current fundamental problems in theoretical physics - the development of quantum information technology, and the problem of quantizing the gravitational field, to name two - require a rethinking of the quantum-classical connection. Graduate students preparing for research careers will find a graduate mechanics course based on this book to be an essential bridge between their undergraduate training and advanced study in analytical mechanics, relativity, and quantum mechanics.

## 作者介绍:

### 目录: CONTENTS

Dedication v

Preface vii

Acknowledgments ix

### PART I INTRODUCTION: THE TRADITIONAL THEORY

1 Basic Dynamics of Point Particles and Collections 3

1.1 Newton's Space and Time 3

1.2 Single Point Particle 5

1.3 Collective Variables 6

1.4 The Law of Momentum for Collections 7

1.5 The Law of Angular Momentum for Collections 8

1.6 "Derivations" of the Axioms 9

1.7 The Work-Energy Theorem for Collections 10

1.8 Potential and Total Energy for Collections 11

1.9 The Center of Mass 11

1.10 Center of Mass and Momentum 13

1.11 Center of Mass and Angular Momentum 14

1.12 Center of Mass and Torque 15

1.13 Change of Angular Momentum 15

1.14 Center of Mass and the Work-Energy Theorems 16

1.15 Center of Mass as a Point Particle 17

1.16 Special Results for Rigid Bodies 17

1.17 Exercises 18

2	Introduction to Lagrangian Mechanics	24
2.1	Configuration Space	24
2.2	Newton's Second Law in Lagrangian Form	26
2.3	A Simple Example	27
2.4	Arbitrary Generalized Coordinates	27
2.5	Generalized Velocities in the $q$ -System	29
2.6	Generalized Forces in the $q$ -System	29
2.7	The Lagrangian Expressed in the $q$ -System	30
2.8	Two Important Identities	31
2.9	Invariance of the Lagrange Equations	32
2.10	Relation Between Any Two Systems	33
2.11	More of the Simple Example	34
2.12	Generalized Momenta in the $q$ -System	35
2.13	Ignorable Coordinates	35
2.14	Some Remarks About Units	36
2.15	The Generalized Energy Function	36
2.16	The Generalized Energy and the Total Energy	37
2.17	Velocity Dependent Potentials	38
2.18	Exercises	41
3	Lagrangian Theory of Constraints	46
3.1	Constraints Defined	46
3.2	Virtual Displacement	47
3.3	Virtual Work	48
3.4	Form of the Forces of Constraint	50
3.5	General Lagrange Equations with Constraints	52
3.6	An Alternate Notation for Holonomic Constraints	53
3.7	Example of the General Method	54
3.8	Reduction of Degrees of Freedom	54
3.9	Example of a Reduction	57
3.10	Example of a Simpler Reduction Method	58
3.11	Recovery of the Forces of Constraint	59
3.12	Example of a Recovery	60
3.13	Generalized Energy Theorem with Constraints	61
3.14	Tractable Non-Holonomic Constraints	63
3.15	Exercises	64
4	Introduction to Hamiltonian Mechanics	71
4.1	Phase Space	71
4.2	Hamilton Equations	74
4.3	An Example of the Hamilton Equations	76
4.4	Non-Potential and Constraint Forces	77
4.5	Reduced Hamiltonian	78
4.6	Poisson Brackets	80
4.7	The Schroedinger Equation	82
4.8	The Ehrenfest Theorem	83
4.9	Exercises	84
5	The Calculus of Variations	88
5.1	Paths in an N-Dimensional Space	89
5.2	Variations of Coordinates	90
5.3	Variations of Functions	91
5.4	Variation of a Line Integral	92
5.5	Finding Extremum Paths	94
5.6	Example of an Extremum Path Calculation	95
5.7	Invariance and Homogeneity	98
5.8	The Brachistochrone Problem	100

5.9 Calculus of Variations with Constraints	102
5.10 An Example with Constraints	105
5.11 Reduction of Degrees of Freedom	106
5.12 Example of a Reduction	107
5.13 Example of a Better Reduction	108
5.14 The Coordinate Parametric Method	108
5.15 Comparison of the Methods	111
5.16 Exercises	113
6 Hamilton's Principle	117
6.1 Hamilton's Principle in Lagrangian Form	117
6.2 Hamilton's Principle with Constraints	118
6.3 Comments on Hamilton's Principle	119
6.4 Phase-Space Hamilton's Principle	120
6.5 Exercises	122
7 Linear Operators and Dyadics	123
7.1 Definition of Operators	123
7.2 Operators and Matrices	125
7.3 Addition and Multiplication	127
7.4 Determinant, Trace, and Inverse	127
7.5 Special Operators	129
7.6 Dyadics	130
7.7 Resolution of Unity	133
7.8 Operators, Components, Matrices, and Dyadics	133
7.9 Complex Vectors and Operators	134
7.10 Real and Complex Inner Products	136
7.11 Eigenvectors and Eigenvalues	136
7.12 Eigenvectors of Real Symmetric Operator	137
7.13 Eigenvectors of Real Anti-Symmetric Operator	137
7.14 Normal Operators	139
7.15 Determinant and Trace of Normal Operator	141
7.16 Eigen-Dyadic Expansion of Normal Operator	142
7.17 Functions of Normal Operators	143
7.18 The Exponential Function	145
7.19 The Dirac Notation	146
7.20 Exercises	147
8 Kinematics of Rotation	152
8.1 Characterization of Rigid Bodies	152
8.2 The Center of Mass of a Rigid Body	153
8.3 General Definition of Rotation Operator	155
8.4 Rotation Matrices	157
8.5 Some Properties of Rotation Operators	158
8.6 Proper and Improper Rotation Operators	158
8.7 The Rotation Group	160
8.8 Kinematics of a Rigid Body	161
8.9 Rotation Operators and Rigid Bodies	163
8.10 Differentiation of a Rotation Operator	164
8.11 Meaning of the Angular Velocity Vector	166
8.12 Velocities of the Masses of a Rigid Body	168
8.13 Savio's Theorem	169
8.14 Infinitesimal Rotation	170
8.15 Addition of Angular Velocities	171
8.16 Fundamental Generators of Rotations	172
8.17 Rotation with a Fixed Axis	174
8.18 Expansion of Fixed-Axis Rotation	176

8.19 Eigenvectors of the Fixed-Axis Rotation Operator	178
8.20 The Euler Theorem	179
8.21 Rotation of Operators	181
8.22 Rotation of the Fundamental Generators	181
8.23 Rotation of a Fixed-Axis Rotation	182
8.24 Parameterization of Rotation Operators	183
8.25 Differentiation of Parameterized Operator	184
8.26 Euler Angles	185
8.27 Fixed-Axis Rotation from Euler Angles	188
8.28 Time Derivative of a Product	189
8.29 Angular Velocity from Euler Angles	190
8.30 Active and Passive Rotations	191
8.31 Passive Transformation of Vector Components	192
8.32 Passive Transformation of Matrix Elements	193
8.33 The Body Derivative	194
8.34 Passive Rotations and Rigid Bodies	195
8.35 Passive Use of Euler Angles	196
8.36 Exercises	198
9 Rotational Dynamics	202
9.1 Basic Facts of Rigid-Body Motion	202
9.2 The Inertia Operator and the Spin	203
9.3 The Inertia Dyadic	204
9.4 Kinetic Energy of a Rigid Body	205
9.5 Meaning of the Inertia Operator	205
9.6 Principal Axes	206
9.7 Guessing the Principal Axes	208
9.8 Time Evolution of the Spin	210
9.9 Torque-Free Motion of a Symmetric Body	211
9.10 Euler Angles of the Torque-Free Motion	215
9.11 Body with One Point Fixed	217
9.12 Preserving the Principal Axes	220
9.13 Time Evolution with One Point Fixed	221
9.14 Body with One Point Fixed, Alternate Derivation	221
9.15 Work-Energy Theorems	222
9.16 Rotation with a Fixed Axis	223
9.17 The Symmetric Top with One Point Fixed	224
9.18 The Initially Clamped Symmetric Top	229
9.19 Approximate Treatment of the Symmetric Top	230
9.20 Inertial Forces	231
9.21 Laboratory on the Surface of the Earth	234
9.22 Coriolis Force Calculations	236
9.23 The Magnetic – Coriolis Analogy	237
9.24 Exercises	239
10 Small Vibrations About Equilibrium	246
10.1 Equilibrium Defined	246
10.2 Finding Equilibrium Points	247
10.3 Small Coordinates	248
10.4 Normal Modes	249
10.5 Generalized Eigenvalue Problem	250
10.6 Stability	252
10.7 Initial Conditions	252
10.8 The Energy of Small Vibrations	253
10.9 Single Mode Excitations	254
10.10 A Simple Example	255

10.11 Zero-Frequency Modes	260
10.12 Exercises	261
PART II MECHANICS WITH TIME AS A COORDINATE	
11 Lagrangian Mechanics with Time as a Coordinate	267
11.1 Time as a Coordinate	268
11.2 A Change of Notation	268
11.3 Extended Lagrangian	269
11.4 Extended Momenta	270
11.5 Extended Lagrange Equations	272
11.6 A Simple Example	273
11.7 Invariance Under Change of Parameter	275
11.8 Change of Generalized Coordinates	276
11.9 Redundancy of the Extended Lagrange Equations	277
11.10 Forces of Constraint	278
11.11 Reduced Lagrangians with Time as a Coordinate	281
11.12 Exercises	282
12 Hamiltonian Mechanics with Time as a Coordinate	285
12.1 Extended Phase Space	285
12.2 Dependency Relation	285
12.3 Only One Dependency Relation	286
12.4 From Traditional to Extended Hamiltonian Mechanics	288
12.5 Equivalence to Traditional Hamilton Equations	290
12.6 Example of Extended Hamilton Equations	291
12.7 Equivalent Extended Hamiltonians	292
12.8 Alternate Hamiltonians	293
12.9 Alternate Traditional Hamiltonians	295
12.10 Not a Legendre Transformation	295
12.11 Dirac's Theory of Phase-Space Constraints	296
12.12 Poisson Brackets with Time as a Coordinate	298
12.13 Poisson Brackets and Quantum Commutators	300
12.14 Exercises	302
13 Hamilton's Principle and Noether's Theorem	305
13.1 Extended Hamilton's Principle	305
13.2 Noether's Theorem	307
13.3 Examples of Noether's Theorem	308
13.4 Hamilton's Principle in an Extended Phase Space	310
13.5 Exercises	312
14 Relativity and Spacetime	313
14.1 Galilean Relativity	313
14.2 Conflict with the Aether	315
14.3 Einsteinian Relativity	316
14.4 What Is a Coordinate System?	318
14.5 A Survey of Spacetime	319
14.6 The Lorentz Transformation	331
14.7 The Principle of Relativity	337
14.8 Lorentzian Relativity	339
14.9 Mechanism and Relativity	340
14.10 Exercises	341
15 Fourvectors and Operators	343
15.1 Fourvectors	343
15.2 Inner Product	346
15.3 Choice of Metric	347
15.4 Relativistic Interval	347
15.5 Spacetime Diagram	349

15.6 General Fourvectors	350
15.7 Construction of New Fourvectors	351
15.8 Covariant and Contravariant Components	352
15.9 General Lorentz Transformations	355
15.10 Transformation of Components	356
15.11 Examples of Lorentz Transformations	358
15.12 Gradient Fourvector	360
15.13 Manifest Covariance	361
15.14 Formal Covariance	362
15.15 The Lorentz Group	362
15.16 Proper Lorentz Transformations and the Little Group	364
15.17 Parameterization	364
15.18 Fourvector Operators	366
15.19 Fourvector Dyadics	367
15.20 Wedge Products	368
15.21 Scalar, Fourvector, and Operator Fields	369
15.22 Manifestly Covariant Form of Maxwell's Equations	370
15.23 Exercises	373
16 Relativistic Mechanics	376
16.1 Modification of Newton's Laws	376
16.2 The Momentum Fourvector	378
16.3 Fourvector Form of Newton's Second Law	378
16.4 Conservation of Fourvector Momentum	380
16.5 Particles of Zero Mass	380
16.6 Traditional Lagrangian	381
16.7 Traditional Hamiltonian	383
16.8 Invariant Lagrangian	383
16.9 Manifestly Covariant Lagrange Equations	384
16.10 Momentum Fourvectors and Canonical Momenta	385
16.11 Extended Hamiltonian	386
16.12 Invariant Hamiltonian	387
16.13 Manifestly Covariant Hamilton Equations	388
16.14 The Klein–Gordon Equation	389
16.15 The Dirac Equation	390
16.16 The Manifestly Covariant N-Body Problem	392
16.17 Covariant Serret–Frenet Theory	399
16.18 Fermi–Walker Transport	401
16.19 Example of Fermi–Walker Transport	403
16.20 Exercises	405
17 Canonical Transformations	411
17.1 Definition of Canonical Transformations	411
17.2 Example of a Canonical Transformation	412
17.3 Symplectic Coordinates	412
17.4 Symplectic Matrix	416
17.5 Standard Equations in Symplectic Form	417
17.6 Poisson Bracket Condition	418
17.7 Inversion of Canonical Transformations	419
17.8 Direct Condition	420
17.9 Lagrange Bracket Condition	422
17.10 The Canonical Group	423
17.11 Form Invariance of Poisson Brackets	424
17.12 Form Invariance of the Hamilton Equations	426
17.13 Traditional Canonical Transformations	428
17.14 Exercises	430

18 Generating Functions	434
18.1 Proto-Generating Functions	434
18.2 Generating Functions of the F1 Type	436
18.3 Generating Functions of the F2 Type	438
18.4 Examples of Generating Functions	439
18.5 Other Simple Generating Functions	441
18.6 Mixed Generating Functions	442
18.7 Example of a Mixed Generating Function	444
18.8 Finding Simple Generating Functions	445
18.9 Finding Mixed Generating Functions	446
18.10 Finding Mixed Generating Functions—An Example	448
18.11 Traditional Generating Functions	449
18.12 Standard Form of Extended Hamiltonian Recovered	451
18.13 Differential Canonical Transformations	452
18.14 Active Canonical Transformations	453
18.15 Phase-Space Analog of Noether Theorem	454
18.16 Liouville Theorem	455
18.17 Exercises	456
19 Hamilton–Jacobi Theory	461
19.1 Definition of the Action	461
19.2 Momenta from the S1 Action Function	462
19.3 The S2 Action Function	464
19.4 Example of S1 and S2 Action Functions	465
19.5 The Hamilton–Jacobi Equation	466
19.6 Hamilton’s Characteristic Equations	467
19.7 Complete Integrals	469
19.8 Separation of Variables	472
19.9 Canonical Transformations	473
19.10 General Integrals	475
19.11 Mono-Energetic Integrals	480
19.12 The Optical Analogy	482
19.13 The Relativistic Hamilton–Jacobi Equation	483
19.14 Schroedinger and Hamilton–Jacobi Equations	483
19.15 The Quantum Cauchy Problem	485
19.16 The Bohm Hidden Variable Model	486
19.17 Feynman Path-Integral Technique	487
19.18 Quantum and Classical Mechanics	488
19.19 Exercises	489

### PART III MATHEMATICAL APPENDICES

A Vector Fundamentals	495
A.1 Properties of Vectors	495
A.2 Dot Product	495
A.3 Cross Product	496
A.4 Linearity	496
A.5 Cartesian Basis	497
A.6 The Position Vector	498
A.7 Fields	499
A.8 Polar Coordinates	499
A.9 The Algebra of Sums	502
A.10 Miscellaneous Vector Formulae	502
A.11 Gradient Vector Operator	504
A.12 The Serret–Frenet Formulae	505
B Matrices and Determinants	508
B.1 Definition of Matrices	508

B.2 Transposed Matrix	508
B.3 Column Matrices and Column Vectors	509
B.4 Square, Symmetric, and Hermitian Matrices	509
B.5 Algebra of Matrices: Addition	510
B.6 Algebra of Matrices: Multiplication	511
B.7 Diagonal and Unit Matrices	512
B.8 Trace of a Square Matrix	513
B.9 Differentiation of Matrices	513
B.10 Determinants of Square Matrices	513
B.11 Properties of Determinants	514
B.12 Cofactors	515
B.13 Expansion of a Determinant by Cofactors	515
B.14 Inverses of Nonsingular Matrices	516
B.15 Partitioned Matrices	517
B.16 Cramer's Rule	518
B.17 Minors and Rank	519
B.18 Linear Independence	520
B.19 Homogeneous Linear Equations	520
B.20 Inner Products of Column Vectors	521
B.21 Complex Inner Products	523
B.22 Orthogonal and Unitary Matrices	523
B.23 Eigenvalues and Eigenvectors of Matrices	524
B.24 Eigenvectors of Real Symmetric Matrix	525
B.25 Eigenvectors of Complex Hermitian Matrix	528
B.26 Normal Matrices	528
B.27 Properties of Normal Matrices	530
B.28 Functions of Normal Matrices	533
C Eigenvalue Problem with General Metric	534
C.1 Positive-Definite Matrices	534
C.2 Generalization of the Real Inner Product	535
C.3 The Generalized Eigenvalue Problem	536
C.4 Finding Eigenvectors in the Generalized Problem	537
C.5 Uses of the Generalized Eigenvectors	538
D The Calculus of Many Variables	540
D.1 Basic Properties of Functions	540
D.2 Regions of Definition of Functions	540
D.3 Continuity of Functions	541
D.4 Compound Functions	541
D.5 The Same Function in Different Coordinates	541
D.6 Partial Derivatives	542
D.7 Continuously Differentiable Functions	543
D.8 Order of Differentiation	543
D.9 Chain Rule	543
D.10 Mean Values	544
D.11 Orders of Smallness	544
D.12 Differentials	545
D.13 Differential of a Function of Several Variables	545
D.14 Differentials and the Chain Rule	546
D.15 Differentials of Second and Higher Orders	546
D.16 Taylor Series	547
D.17 Higher-Order Differential as a Difference	548
D.18 Differential Expressions	548
D.19 Line Integral of a Differential Expression	550
D.20 Perfect Differentials	550

- D.21 Perfect Differential and Path Independence 552
- D.22 Jacobians 553
- D.23 Global Inverse Function Theorem 556
- D.24 Local Inverse Function Theorem 559
- D.25 Derivatives of the Inverse Functions 560
- D.26 Implicit Function Theorem 561
- D.27 Derivatives of Implicit Functions 561
- D.28 Functional Independence 562
- D.29 Dependency Relations 563
- D.30 Legendre Transformations 563
- D.31 Homogeneous Functions 565
- D.32 Derivatives of Homogeneous Functions 565
- D.33 Stationary Points 566
- D.34 Lagrange Multipliers 566
- D.35 Geometry of the Lagrange Multiplier Theorem 569
- D.36 Coupled Differential Equations 570
- D.37 Surfaces and Envelopes 572
- E Geometry of Phase Space 575
- E.1 Abstract Vector Space 575
- E.2 Subspaces 577
- E.3 Linear Operators 578
- E.4 Vectors in Phase Space 580
- E.5 Canonical Transformations in Phase Space 581
- E.6 Orthogonal Subspaces 582
- E.7 A Special Canonical Transformation 582
- E.8 Special Self-Orthogonal Subspaces 583
- E.9 Arnold's Theorem 585
- E.10 Existence of a Mixed Generating Function 586
- References 588
- Index 591
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